Stochastic Processes in Cell Biology I: Notes and Corrections

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August 23, 2024

Chapter 7
Active transport

p. 551. Delete the insert “$F_j(t)$” at the end of the first para in Section 7.4.

p. 551. Delete the sentence *“It remains to determine the distribution $F_j(t)$ for a given target”* at the start of the second para of Section 7.4

p. 552. In the sentences above and below equation (7.4.2) replace *”inter-arrival time”* by *“first-arrival time”*. In the derivation of the renewal equation for the Binomial moments we condition on the first (or next) arrival time with distribution $\mathcal{F}_j(t)$. Suppose, instead, that $\mathcal{F}_j(t)$ denotes the inter-arrival time density of packets to the jth target The relationship between the conditional FPT density $f_j(t)$ of a single search-and-capture process and $\mathcal{F}_j(t)$ is more complicated than the single target case. This is due to the fact that the arrival event of the next packet to the j-th target could occur after an arbitrary number of deliveries to other targets. It follows that (ignoring refractory periods)

$$\begin{aligned}\mathcal{F}_j(t) &= \pi_j f_j(t) + \pi_j \sum_{k\neq j} \pi_k \int_0^t f_k(\tau) f_j(t-\tau) d\tau \\ &\quad + \pi_1 \sum_{k,k'\neq j} \pi_k \pi_{k'} \int_0^t \int_0^{t-\tau} f_k(\tau) f_{k'}(\tau') f_j(t-\tau-\tau') d\tau' d\tau + \ldots \end{aligned} \tag{7.1}$$

Laplace transforming both sides using the convolution theorem gives

$$\widetilde{\mathcal{F}}_j(s) = \pi_j \widetilde{f}_j(s) + \pi_j \sum_{k\neq j} \pi_k \widetilde{f}_j(s) \widetilde{f}_k(s) + \pi_j \sum_{k,k'\neq j} \pi_k \pi_{k'} \widetilde{f}_j(s) \widetilde{f}_k(s) \widetilde{f}_{k'}(s) + \ldots \tag{7.2}$$

Summing the geometric series leads to the closed expression

$$\widetilde{\mathcal{F}}_j(s) = \frac{\pi_j \widetilde{f}_j(s)}{1 - \sum_{k\neq j} \pi_k \widetilde{f}_k(s)}. \tag{7.3}$$

Note, in particular, that

$$\int_0^\infty \mathcal{F}_j(t)dt = \widetilde{\mathcal{F}}_j(0) = \frac{\pi_j \widetilde{f}_j(0)}{1-\sum_{k\neq j}\pi_k \widetilde{f}_k(0)}. = \frac{\pi_j}{1-\sum_{k\neq j}\pi_k} = 1 \tag{7.4}$$

as required. We have used the fact that $\widetilde{f}_j(0) = \int_0^\infty f_j(t)dt = 1$ for all $j = 1,\ldots,N$.

p. 556. Delete *"(We keep the symbol r as the resetting rate.)"* below equation (7.4.19a):

p. 557. Delete *"without resetting"* below equation (7.4.21)

p. 558: Caption of Fig. 7.21. Change *"as a function of resetting radius x_r for $d = 1,2,3$ and $r = 0.1,10$"* to *"as a function of x_0 for $d = 1,2,3$"*